Measurements indicated that the period, width, and amplitude of the current pulses decreased as the voltage was increased, with minor fluctuations occurring in their values. Changing the liquid pressure had a relatively small effect on the amplitude and width of the current pulses, but it had a marked effect on the pulsing period. It was also observed that the minimum spraying potential decreased as the liquid pressure was increased.

As the voltage was increased, the period-to-width ratio decreased to a value of about three. At this point, the d.c. spray-current level suddenly jumped to a new value, indicating an essentially constant current. The current pulses themselves disappeared. As the voltage was further increased, the new d.c. level also increased, and the current pulses reappeared. More than one set of current pulses was now present, not necessarily of the same amplitude and frequency, and sprays were located at different points on the periphery of the capillary tip. Increasing the voltage caused additional current pulses, smaller in amplitude and width, until the current looked like a jumble of noise.

Using a larger diameter capillary or increased spacing between capillary and grounded electrode gave results similar to the foregoing but translated to higher voltage values.

The naturally pulsed spraying, which was characteristic of low voltages and low liquid pressures, made the use of a time-of-flight mass spectrometer attractive for determining specific-charge spectra. A positive voltage was applied to an electrode at the entrance of the instrument so that sprayed droplets entered the drift space only when a negative control pulse was applied to the electrode. The control pulse was synchronized with the current pulse produced by the spraying glycerine, and its relative position could be varied at will. The drift time, and hence the specific-charge values, were accurately measured by using a narrow control pulse.

A series of measurements on the pulsed spraying of glycerine at low voltages indicated that the specific charges produced at the peaks of the current pulses were lower and more widely spread in values than at the beginning and end of the current pulses. A typical specific-charge range was 0.2 to 1.8 coul/kg with a few random values up to about 10 coul/kg.

The various forementioned measurements show that the disruptive effects of electric fields are important in the field emission of macroscopic liquid droplets. Not only the initial instability of the liquid surface, which generates the charged particles, but also the stability of a preformed charged particle injected into an accelerating field is of interest. For a rigid sphere of radius a in a uniform electric field E, it can be shown that equating surface-tension pressure to maximum electric stress on the sphere (the point of instability) results in

$$\frac{9}{4}aE^2 + \frac{a^2\rho}{2\epsilon_0}\left(\frac{q}{m}\right) + \frac{a^3\rho^2}{36\epsilon_0^2}\left(\frac{q}{m}\right)^2 - \frac{\gamma}{\epsilon_0} = 0 \qquad (1)$$

where ρ is the mass density, m the mass, γ the coefficient of surface tension, and q the total charge on the sphere. The following two cases are immediately of interest.

Case 1: $q = 0$. In this case, Eq. (1) simplifies to

$$\tfrac{9}{4}\,aE^2 = \gamma/\epsilon_0$$

or

$$E(a)^{1/2} = \tfrac{2}{3}(\gamma/\epsilon_0)$$

This becomes, for water with $\gamma = 58$ dynes/cm, $E(a)^{1/2} = 5400$ v/cm$^{1/2}$. Experimental measurements on the bursting of soap bubbles show that $E(a)^{1/2} = 3600$ v/cm$^{1/2}$. The difference here may be attributed to the fact that the soap bubbles depart from the spherical shape before becoming unstable. This occurs because the prolate spheroidal shape is one of less stored energy than the spherical shape in this case.

Case 2: In order for the quadratic equation (1) to have positive solutions for E (the only meaningful solutions), it is found that

$$\gamma > \frac{a^3\rho^2}{36\epsilon_0}\left(\frac{q}{m}\right)^2$$

which is the well-known Rayleigh limit for charged droplets.

As a final example of the instability of a charged particle in a uniform electric field, suppose $a = 10^{-8}$ m, $(q/m) = 10^3$ coul/kg, $\rho = 10^3$ kg/m^3, and $\gamma = 0.1$ newtons/m. These values are typical for the organic liquids in present use. Then $E = 5.4 \times 10^8$ v/m, which value is high. Hence, in order to use an accelerating potential of 10^6 v, the spacing should probably be greater than 0.3 cm. For a higher specific charge with the same droplet, this distance would have to be greater.

Gaseous Nuclear Rocket with MHD Vortex Fuel Containment

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I. Introduction

THE potentialities of a nuclear rocket are well known,[1-3] but many difficult and fundamental problems remain to be solved before an attempt is made to build a practical device. One of the major problems is the separation of the gaseous fuel (for example, uranium 235) from the propellant (hydrogen). It was this particular aspect of the problem which motivated the previous study by Gross and Kessey.[5] In that study, a hydromagnetically driven vortex separation system is analyzed with primary attention given to the magnetohydrodynamics of a cylindrical system.

In the present note, we examine some potentialities and limitations of this concept for a practical nuclear rocket system. We determine the size, flow limitations, composition of the exhaust gas, electrical power, reactor power, etc. required by a hydromagnetic vortex, gaseous nuclear rocket reactor. The pertinent analytical results of Refs. 4 and 5 are applied to a practical system. The simultaneous requirements for nuclear criticality, adequate fuel-propellant separation, input composition to maintain a steady state, and limits on electrical powerplant weight form the crux of this analysis of a magnetohydrodynamic vortex nuclear rocket.

II. Physical Conditions for Steady Nuclear Critical Flow

The relevant hydromagnetic equations that we have employed are lengthy and can be found in Refs. 4–6. The notation used here is identical to that described in these papers. There are several simultaneous physical constraints in the proposed system which must be satisfied when the relevant equations [for example, (5, 32, 38, 47, and 48) of Ref. 5, or (1, 8, 10, and 11) of Ref. 6] are evaluated for the gaseous nuclear rocket. These constraints are briefly:

1) The radial mass flow rate (i.e., G_0 or radial Reynolds number R_N) must be such that the solutions of the equations yield positive values for the relative composition. There is a limit on the radial Reynolds number for which n_U/n_{0U} is positive. The same type of limit on radial mass flow was

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Table 1 Physical properties of the plasma (mks units) at 1 ev[a]

	$\zeta_1 = 0.1$	$\zeta_1 = 0.3$	$\zeta_1 = 0.5$
Volume, m^3	11.2	10.7	8.55
Number density of U^{235}, particles/m^3	3.41×10^{24}	3.58×10^{24}	4.48×10^{24}
Number density of H	3.41×10^{26}	3.58×10^{26}	4.48×10^{26}
Plasma density, kg/m^3	1.90	2.00	2.50
Pressure $\cong \langle n_H \rangle kT$, nt/$m^2$	5.81×10^7	5.44×10^7	5.82×10^7
(atm)	(510)	(535)	(575)
Electrical conductivity, σ mhos/m	1×10^3	$1 + 10^3$	1×10^3
Coefficient of diffusion, D_{HU}, m^2/sec	5×10^{-4}	5×10^{-4}	5×10^{-4}
Absolute coefficient of viscosity η, kg/m-sec	9.50×10^{-4}	1.00×10^{-3}	1.25×10^{-3}
M Hartmann number	1020	1000	900
R_N radial Reynolds number	−55.0	−115	−220
S_N Schmidt number	1.00	1.00	1.00
ϵ_1 $(m_u\eta^2/2\rho^2 r_0^2 kT_0)$	3.17×10^{-13}	3.17×10^{-13}	4.17×10^{-13}
K $(\rho\phi_0/B_0\eta)$	1×10^5	1×10^5	1×10^5

[a] Temperature, $T_0 = 1$ ev; mass of uranium = 15 kg.

found by Kerrebrock and Meghreblian.[7] Negative values of n_U are, of course physically impossible and most probably imply that a steady-state solution with the assumed R_N is unobtainable.

2) The chemical composition of the gaseous cavity must be such that nuclear criticality is obtained in the H-U^{235} mixture. Although the composition varies with radial position, on the basis of the criticality results given in Ref. 5 we have assumed that the average composition ratio $\langle n_U/n_H \rangle$ in the cavity lies between 10^{-3} and 10^{-2}. In the absence of more accurate nonhomogeneous criticality calculations, we consider this ratio to be sufficiently accurate.

3) In order that the fuel be satisfactorily confined to the reactor cavity, the inner "wall" composition $(n_U/n_H)_{\zeta=\zeta_1}$ must be less than 10^{-3}. Thus, it is required that less than one part of uranium per thousand parts of hydrogen be present in the exhaust gas. This ratio is believed to be practical as shown by Bussard and DeLauer.[1] It will be shown that, in fact, this ratio can be as small as 10^{-4} in some cases that are detailed in a later section. In addition, for a steady state, the input composition (near the outer cylinder) must equal the exhaust composition (at the inner cylindrical wall).

4) In principle, the composition of the gas at the inner wall, as discussed previously in item 3, can be made as small as desired by applying more electrical power to spin the gas at a higher rate. That is to say, increasing K (or the plasma drift forces over the viscous forces), and consequently the electrical power requirement, enhances the radial concentration gradient of the lighter species. However, the source of electrical power for the MHD spinning requires weight that must be minimized in any flight system. As a matter of compromise (and not optimization) we shall take the azimuthal kinetic power $(MV_\theta^2/2)$ to be less than 10% of the longitudinal rocket exhaust power. Although this may not permit thrust to weight ratio in excess of unity, it does permit thrust to weight ratios that are of great interest for high-energy upper stages. A decrease in electrical spin power available permits an increase in thrust to weight ratio, but at the expense of losing more U^{235} from the reactor. Since realistic weight figures require detailed design studies, it does not seem advisable for us, at this stage, to further attempt to optimize this important design parameter.

An approximate expression that satisfies the criterion on radial Reynolds number can be easily developed, and it is

$$(R_N)_{\max} \cong \frac{n_{0U}/n_{0H}}{\alpha}\left(\frac{1-\alpha}{1+n_{0U}/n_{0H}}\right)\frac{1}{S_N \ln\zeta_1} \tag{1}$$

Equation (1) is a quick guide to obtaining the maximum permissible R_N, but of course Eq. (11) of Ref. 6 gives the exact value.

As mentioned in item 2, the average relative composition of interest based on criticality calculations is $10^{-2} < \langle n_U/n_H \rangle < 10^{-3}$. This is defined as

$$\left\langle \frac{n_H}{n_U} \right\rangle \equiv \frac{\iiint_V n_H dV}{\iiint_V n_U dV}$$

which can be integrated[6] to give

$$\left\langle \frac{n_H}{n_U} \right\rangle = \frac{\dfrac{\alpha + (n_{0U}/n_{0H})}{1-\alpha}\left\{\epsilon_1 K^2 \mathfrak{R}^2(\zeta_1, M, R_N) I_1 + S_N R_N \left[\dfrac{1+(n_{0U}/n_{0H})}{\alpha+(n_{0U}/n_{0H})}\right] I_2\right\} + \dfrac{1-\zeta_1^2}{2}}{\alpha\left[\dfrac{\alpha+(n_{0U}/n_{0H})}{\alpha-1}\right]\left\{\epsilon_1 K^2 \mathfrak{R}^2(\zeta_1, M, R_N) I_1 + S_N R_N \left[\dfrac{1+(n_{0U}/n_{0H})}{\alpha+(n_{0U}/n_{0H})}\right] I_2\right\} + \dfrac{1-\zeta_1^2}{2}} \tag{2}$$

Thus, for any values of the parameters involved, the value of n_{0U}/n_{0H}, the concentration ratio at $\zeta = 1$, may be determined from Eq. (2) so as to yield the required average number density ratio.

Lastly, for steady separation of species, the concentration ratio of the injected plasma must equal that of the mixture extracted at the rocket nozzle, as mentioned in item 3 previously. That is,

$$(n_H/n_U)_{in} = (n_H/n_U)_{out}$$

For steady state, the composition of the inflow plasma is found to be

$$\left(\frac{n_H}{n_U}\right)_{in} = \frac{\epsilon_1 K^2 \mathfrak{R}^2 f_p(\zeta = \zeta_1') + S_N R_N \alpha_p \ln\zeta_1 + \dfrac{1-\alpha}{1+n_{0U}/n_{0H}}}{\dfrac{1-\alpha}{\alpha+n_{0U}/n_{0H}} - \alpha[\epsilon_1 K^2 \mathfrak{R}^2 f_p(\zeta = \zeta_1) + S_N R_N \alpha_p \ln\zeta_1]} \tag{3}$$

Equation (3) explicitly gives the relationship between the compositions of the injected plasma and the mixture at the outer electrode. The transition from the inflow state to the state at the outer radius involves a mixing mechanism that is not well understood but has been discussed by Kerrebrock.[7]

III. Reactor Size and Physical Properties

We must determine the physical properties of the gaseous medium within the reactor, as well as set a size and energy level of the reactor. We begin by choosing the stagnation temperature of the exhaust gas to be 1 ev (11,600°K) which will yield a specific impulse of about 3000 sec.[5] This is a sufficiently interesting specific impulse, yet at a temperature that is believed to be practical within heat-transfer tolerances of the proposed system. We have not carried out a parametric study but, by judicious choice, we have tried to operate within a range of values which produces a large gain in per-

formance and yet which lies within the limits of engineering practice.

A. Reactor dimensions

From Safonov[8] we choose a reactor radius of 1 m, and, to minimize the reactor volume for nuclear criticality, we select a cylindrical height of 1.8 m.[5] Again, from Ref. 8, we require approximately 15 kg of U^{235} for criticality. This gives a uranium 235 number density of about $\langle n_U \rangle \approx 3 \times 10^{24}$ particles/m³ depending slightly upon the radius of the inner cylinder. If we assume that $\langle n_U/n_H \rangle = 10^{-2}$, then the average number density of hydrogen is $\langle n_H \rangle \cong 3 \times 10^{26}$ particles/m³. The volume of the reactor cavity is about 10 m³ and contains about 20 kg (or 44 lb) of gaseous matter. The static pressure of the gaseous mixture at 1 ev is given by

$$\bar{p} = kT_0 \Sigma n_i$$

and under these conditions we obtain a pressure of about 500 atm. Although this is quite high, the requirements of nuclear criticality and high temperature necessitate a pressure in excess of 100 atm.

B. Transport properties

Since the gas within the reactor is primarily hydrogen (99%), we shall neglect the presence of uranium and compute the electrical conductivity and the coefficient of ordinary diffusion for hydrogen at 1 ev and 500 atm. From the calculations by Eisen and Gross,[9] the electrical conductivity σ is approximately 10^3 mho/m. From kinetic theory, the coefficient of diffusion is given by

$$D_{HU} = (1/2n_0 Q_{HU})(kT_0/\gamma m^*)^{1/2} \tag{4}$$

where m^* is the reduced mass and Q_{HU} the elastic collision cross section between hydrogen and uranium. Since the gaseous material consists primarily of neutral monatomic hydrogen, we choose $Q_{HU} \cong 10^{-15}$ cm². Then Eq. (4) gives $D_{HU} \cong 5 \times 10^{-4}$m²/sec. The viscosity is simply related to D_{HU} by $\eta = \rho D_{HU}$, whereas the density ρ is given by

$$\rho = m_H \langle n_H \rangle + m_U \langle n_U \rangle \tag{5}$$

The fluid density is about 2 kg/m³, and hence $\eta \cong 10^{-3}$ kg/m-sec. From these properties, M, R_N, K, S_N, and ϵ_1 are computed for an impressed axial magnetic field of 1 weber/m² and radial voltage difference of 50 v. The results are summarized in Table 1.

IV. Reactor Power Requirements

Owing to the fact that the ratio of the concentration of propellant to that of the fuel upon entering the reactor must equal the exhaust composition [which is $(n_U/n_H) \cong 10^{-4}$], it is assumed that the through-flow material consists of hydrogen only. Consequently, the enthalpy change of uranium 235 between the inflow and outflow points is neglected compared to that of hydrogen. From conservation of energy between the inflow and outflow points,

$$H_{in} + KE_{in} = H_{out} + KE_{out} + Q \tag{6}$$

where H is the specific enthalpy, KE the average kinetic energy per unit mass, and Q the nuclear reactor energy input per unit mass. We assume that $KE_{in} = 0$. Now,

$$KE_{out} \cong \tfrac{1}{2}\langle V_\theta^2 \rangle$$

and it is easily shown[6] that the radial kinetic energy is negligible.

The exit kinetic energy is

$$KE_{out} = \frac{K^2(\eta/\rho r_0)^2 \Re^2(\zeta_1, M, R_N)}{1 - \zeta_1^2} \times \left\{ \frac{a_1^2}{2(m+1)} [1 - \zeta_1^{2m+2}] - a_2^2 \ln \zeta_1 - \frac{1}{4}(1 - \zeta_1^4) + \frac{2a_1 a_2}{m+1}[1 - \zeta_1^{m+1}] - \frac{2a_1}{m+3}[1 - \zeta_1^{m+3}] - a_2[1 - \zeta_1^2] \right\} \tag{7}$$

To obtain the enthalpy change between the inflow molecular hydrogen and the outflow monatomic hydrogen and ions, consider the chemical reaction

$$H_2 \rightarrow \nu_H H + \nu_{H_2} H_2 + \nu_{H^+} H^+ + \nu_e e^- \tag{8}$$

where ν_i is the number of moles of the ith component at the reactor exit per mole of input diatomic hydrogen. For conservation of protons and electric charge, we require

$$\nu_H + 2\nu_{H_2} + \nu_{H^+} = 2 \tag{9}$$

and

$$\nu_{H^+} = \nu_e \tag{10}$$

If we assume chemical equilibrium and use the thermochemical data of Refs. 9 and 10, we find, at 500 atm, 1 ev,[6]

$$\langle n_H \rangle = 3 \times 10^{26} \text{ part/m}^3$$
$$\langle n_e \rangle = 3 \times 10^{24} \text{ part/m}^3$$
$$\langle n_{H_2} \rangle = 1.5 \times 10^{23} \text{ part/m}^3$$

The principal energy sink is the dissociation of the diatomic hydrogen. The enthalpy difference between the cold (room temperature) hydrogen and the rocket exhaust at 1 ev is found to be 4.75×10^8 joules/kg.

The reactor power is obtained from

$$P_R = WQ = W[H_{in} - H_{out} - KE_{out}] \tag{11}$$

where W is the radial mass flow rate. Under conditions of 500 atm and reactor temperature of 11,600°K, the reactor power P_R is 4.75×10^5 kw if the radial mass flow rate is 1 kg/sec. The exact values of relative compositions and reactor power for three reactors considered are summarized in Table 2.

V. Results and Comments

For the system and the physical properties as described, we find the maximum permissible radial mass flow rate compatible with the system constraints discussed in Sec. II.

Table 2 Summary of reactor conditions[a]

	$\zeta_1 = 0.1$	$\zeta_1 = 0.3$	$\zeta_1 = 0.5$
Moles of H/mole of input H_2, ν_H	2.00	2.00	2.00
Moles of H_2/mole of input H_2, ν_{H_2}	1.02×10^{-3}	1.07×10^{-3}	1.15×10^{-3}
Moles of H^+ or e/mole of input H_2, $\nu_{H^+} = \nu_e$	2.00×10^{-2}	1.95×10^{-2}	1.87×10^{-2}
Average density of H $\langle n_H \rangle$, particles/m³	3.41×10^{26}	3.58×10^{26}	4.48×10^{26}
Average density of H^+ or e^- $\langle n_{H^+} \rangle = \langle n_e \rangle$, particles/m³	3.41×10^{24}	3.5×10^{24}	4.27×10^{24}
Average density of H_2 $\langle n_{H_2} \rangle$, particles/m³	1.74×10^{23}	1.92×10^{23}	2.62×10^{23}
Total enthalpy change, $H_{out} - H_{in}$, joules/kg	4.75×10^8	4.74×10^8	4.73×10^8
Average kinetic energy KE_{out}, joules/kg	1.23×10^3	2.68×10^3	5.90×10^3
Radial mass flow W, kg/sec	1.18	2.55	5.67
Reactor power P_R, KW	5.60×10^5	1.21×10^6	2.69×10^6

[a] $T = 11{,}600$°K; $T_1 = 300$°K.

Table 3 Performance characteristics of a gaseous nuclear rocket[a]

	$\zeta_1 = 0.1$	$\zeta_1 = 0.3$	$\zeta_1 = 0.5$
Electrical power for MHD vortex, kw	3.309	17.4	87.9
Radial electric current, amp	66.2	348	1760
Average rotational velocity, m/sec	55.5	71.4	99.9
Average density ratio $\langle n_H/n_U \rangle$	54.8	107.6	147.1
Density ratio at inner wall (n_H/n_U)	2.2×10^3	1.844×10^4	2.042×10^4
Maximum permissible R_N	−60	−116	−202
Reynolds number used	−55	−115	−200
Mass flow rate, lb/sec	2.61	5.63	12.5
Thrust, lb	7830	16,890	37,500

[a] Specific impulse = 3000 sec; $r_0 = 1$ m; temperature = 1 ev; $n_{0U}/n_{0H} = 1.5$; $\phi_0 = 50$ v; $B_0 = 1$ weber/m²; mass of $U^{235} = 15$ kg.

For an axial magnetic field strength of 1 weber/m² and radial voltage difference of 50 v maintained between the concentric cylinders, we find the results for maximum permissible mass flow rate shown in Fig. 1. These results unfortunately correspond to very low propellant flow rates and constitute a major limitation to the thrust obtainable with a single vortex system. Table 3 summarizes the rocket characteristics.

The major limitation on this system appears to be the low value of radial mass flow resulting in low total thrust for a single vortex system. Even for moderate electrical power requirement, the theoretical containment of the uranium fuel is quite good: the concentration of U^{235} in the rocket exhaust is of the order of one-ten-thousandth of the concentration of monatomic hydrogen.

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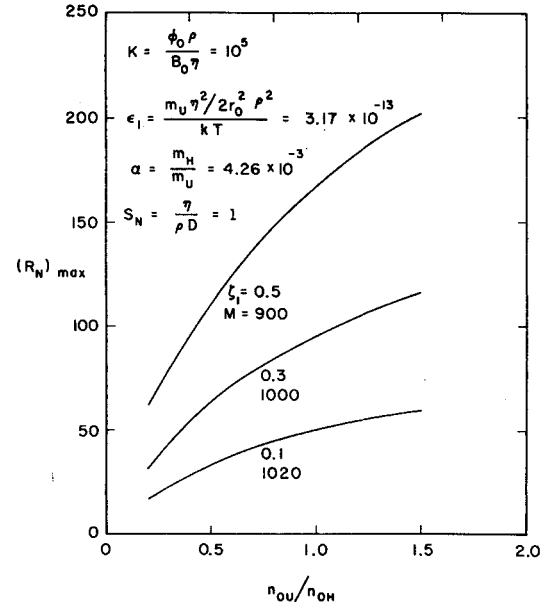

Fig. 1 Variation of the maximum permissible radial Reynolds number with ζ_1 vs boundary composition ratio.

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A Transformation for Wake Analyses

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LAMINAR near wakes are receiving much emphasis at present. It is the purpose of this note to show that analyses that already have been made for turbulent near wakes are of value in studying laminar near wakes by means of a simple transformation.

The equations for both turbulent and laminar jet mixing have been developed.[1] From those results, it can be observed that half-infinite two-dimensional laminar jet mixing is governed by equations that are identical to the turbulent case, if the turbulent similarity parameter σ is replaced by an equivalent laminar parameter and if the laminar viscosity is assumed constant. The transformation condition (turbulent-to-laminar or vice versa) which must be satisfied is

$$\sigma = [Ux/\nu\alpha]^{1/2}/2 \qquad (1)$$

where U is the adjacent potential velocity, x is the mixing region length, ν is the viscosity, and α is a reference perturbation velocity factor ($1.0 \leq \alpha \leq 2.0$).

With the aid of this transformation, the tested and proved turbulent theories and solutions (e.g., Ref. 2) for situations in which the jet mixing is controlling (e.g., the base flow or near-wake problem) may be transferred to laminar constant viscosity conditions. Reference 3 applies as well for the laminar constant viscosity case as the turbulent case when the transformation law is used. If the viscosity cannot be considered constant in the mixing region, then results obtained by using this transformation must be considered as a first approximation.

As an example of application of the transformation, Fig. 1 shows the dimensionless jet-boundary-streamline velocity vs

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